

STABILITY ANALYSIS OF ECOLOGICAL MATHEMATICAL MODEL OF OXYGEN-PHYTOPLANKTON-ZOOPLANKTON

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ABSTRACT

Aquatic ecosystems are strongly influenced by interactions among dissolved oxygen concentration, phytoplankton, and zooplankton populations. Phytoplankton play a fundamental role as primary producers through photosynthesis, while zooplankton act as consumers regulating phytoplankton growth. In this study, a mathematical model describing the interaction between oxygen concentration, phytoplankton, and zooplankton is developed and analyzed. The model is formulated as a system of nonlinear ordinary differential equations. Analytical investigations include the positivity and boundedness of solutions, equilibrium analysis, and local stability analysis using the Jacobian matrix and the Routh–Hurwitz stability criterion. Several equilibrium points are obtained, including the trivial equilibrium, boundary equilibria, and an interior equilibrium representing coexistence among the three ecological components. Numerical simulations are performed using MATLAB to illustrate the dynamical behavior of the system. The results indicate that the stability of the system depends on ecological parameters such as phytoplankton growth rate, zooplankton mortality, and oxygen production. The analytical and numerical results demonstrate that the ecosystem can reach stable states under certain parameter conditions.



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1. Introduction

Aquatic ecosystems such as oceans, rivers, and lakes consist of complex interactions between biological organisms and environmental factors. One of the key components in aquatic ecosystems is plankton, which plays an essential role in maintaining ecological balance and supporting aquatic food webs. Plankton can generally be classified into phytoplankton and zooplankton. Phytoplankton function as primary producers capable of photosynthesis and oxygen production, whereas zooplankton serve as consumers that feed on phytoplankton and regulate their population growth [1]. The interaction between these two groups significantly affects nutrient cycles and ecosystem productivity. Another crucial factor in aquatic ecosystems is dissolved oxygen concentration. Oxygen concentration in water is influenced by several biological and physical processes including photosynthesis, respiration of aquatic organisms, decomposition of organic matter, and gas exchange with the atmosphere [2]. An imbalance between oxygen production and consumption may lead to environmental problems such as hypoxia or algal blooms [3].

Mathematical modeling provides a useful framework for understanding complex ecological interactions and predicting ecosystem responses to environmental changes. Mathematical models can represent nonlinear interactions between biological and environmental variables [4]. Through stability analysis, it is possible to determine whether an ecological system tends to reach a stable equilibrium or experience oscillatory or unstable behavior [5]. Previous studies have explored plankton dynamics using mathematical models. May investigated the stability of ecological systems using nonlinear models [6]. Huisman and Weissing analyzed biodiversity in plankton communities through oscillatory dynamics [7]. Edwards studied the role of zooplankton mortality in plankton population dynamics [8], while Steele introduced functional responses in marine ecosystem models [9]. More recent studies have also incorporated oxygen dynamics and environmental variability into plankton models [10], [11].

Motivated by these studies, this research develops and analyzes a mathematical model describing the interaction between dissolved oxygen, phytoplankton, and zooplankton populations. The objective is to determine the equilibrium points of the system and analyze their stability properties.

2. Research Methods

This study employs a theoretical and analytical approach to investigate the stability of a mathematical model describing the interaction between dissolved oxygen, phytoplankton, and zooplankton in aquatic ecosystems. The research is based on literature review, mathematical modeling, stability analysis, and numerical simulation. Relevant scientific sources related to plankton ecology, ecological modeling, and differential equation systems were reviewed to establish the theoretical framework of the study. The ecological model is formulated as a system of nonlinear ordinary differential equations representing the dynamics of dissolved oxygen concentration $c(t)$, phytoplankton population $p(t)$, and zooplankton population $z(t)$. The formulation is based on several biological assumptions, including oxygen production through phytoplankton photosynthesis, oxygen consumption through respiration processes, predation of phytoplankton by zooplankton, and natural mortality of both populations.

The analysis is conducted through several steps. First, equilibrium points of the model are determined by solving the steady-state conditions of the system. Second, local stability of each equilibrium point is analyzed using linearization through the Jacobian matrix and eigenvalue analysis. The Routh–Hurwitz criterion is applied to determine stability conditions of the characteristic polynomial. Finally, numerical simulations are performed using MATLAB to illustrate the dynamic behavior of the system and to support the analytical results. Parameter values used in the simulations are obtained from previously published studies.

3. Results And Discussion

3.1. Mathematical Model of the Oxygen-Phytoplankton-Zooplankton System

The mathematical model describes the interaction between dissolved oxygen concentration, phytoplankton population, and zooplankton population in an aquatic ecosystem. The model is developed based on several biological assumptions, including oxygen production through phytoplankton photosynthesis, oxygen consumption due to respiration processes, and trophic interactions between phytoplankton and zooplankton. Let $c(t)$, $p(t)$, and $z(t)$ denote the concentrations of oxygen, phytoplankton population density, and zooplankton population density at time t , respectively.

Based on the formulated assumptions, the ecological dynamics are represented by the following nonlinear differential equations:

$$\begin{aligned}\frac{dc}{dt} &= \frac{Ac_0p}{c+c_0} - \frac{\delta cp}{c+c_2} - \frac{vcz}{c+c_3} - mc \\ \frac{dp}{dt} &= \left(\frac{Bc}{c+c_1} - \gamma p \right) p - \frac{ap^2z}{ahp^2+p+g} - \sigma p\end{aligned}\quad (1)$$

$$\frac{dz}{dt} = \left(\frac{\eta c^2}{c^2+c_4^2} \right) \cdot \frac{ap^2z}{ahp^2+p+g} - \mu z$$

with initial conditions:

$$c(0) > 0, \quad p(0) > 0, \quad z(0) > 0 \quad (2)$$

All the parameters are positive due to their biological meaning and are described in **Table 1**.

Table 1. Description of biologically meaningful parameters

Parameter	Description
A	Effect of environmental factors on the rate of oxygen production due to the photosynthesis of phytoplankton
δ	Maximum per capita phytoplankton respiration rate
v	Maximum per capita zooplankton respiration rate
m	Rate of oxygen loss due to the biochemical reaction in a marine ecosystem
B	Maximum phytoplankton per capita growth rate in the high oxygen limit
$c_i, i = 0,1,2,3,4$	Half saturation constant of the corresponding processes
γ	Mortality rate due to intraspecific competition among individual phytoplankton
a	Maximally achievable search rate of zooplankton
h	Handling time of zooplankton
g	Half saturation constant
σ	Natural mortality rate of phytoplankton. It is assumed that $B > \sigma$
$\eta \in (0,1)$	Maximum feeding efficiency
μ	Mortality rate of zooplankton

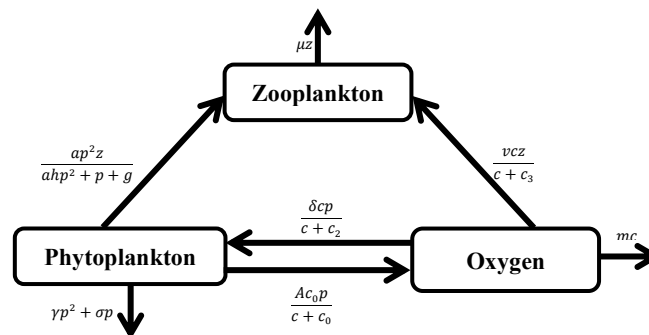


Figure 1. Graphical scheme representing interactions among oxygen, phytoplankton, and zooplankton

Description of system (1), such as:

1. Phytoplankton represents the primary producer in the aquatic ecosystem and plays an essential role in oxygen production through photosynthesis. The population dynamics of phytoplankton are influenced by its intrinsic growth and mortality, represented by the term $\gamma p^2 + \sigma p$. In addition, phytoplankton serves as the main food source for zooplankton, where the grazing process is described by the nonlinear functional response $\frac{ap^2z}{ahp^2+p+g}$. Phytoplankton also interacts with dissolved oxygen through oxygen-dependent processes represented by $\frac{\delta cp}{c+c_2}$, while contributing to oxygen production via term $\frac{Ac_0p}{c+c_0}$.
2. Zooplankton acts as a consumer in the ecosystem and feeds primarily on phytoplankton. The grazing interaction between zooplankton and phytoplankton follows a nonlinear functional response given by $\frac{ap^2z}{ahp^2+p+g}$, which reflects the saturation effect in predation as phytoplankton density increases. The growth of zooplankton depends on the availability of phytoplankton as food, while the population decreases due to natural mortality represented by the term μz . Zooplankton also contributes to oxygen consumption through respiration processes.
3. Dissolved oxygen represents an important environmental variable that supports biological activities within the aquatic ecosystem. Oxygen is mainly produced through the photosynthetic activity of phytoplankton,

represented by the term $\frac{Ac_0p}{c+c_0}$. On the other hand, oxygen is consumed by zooplankton respiration, modeled by the term $\frac{vcz}{c+c_3}$. In addition, dissolved oxygen may be lost from the system due to environmental processes such as diffusion or biochemical consumption, represented by the term mc . The level of dissolved oxygen also influences phytoplankton dynamics through the interaction term $\frac{\delta cp}{c+c_2}$.

3.2. Equilibrium Points of the Model

System (1) has the following equilibrium points:

1. Trivial equilibrium point $E_0(0,0,0)$ corresponding to depletion of oxygen and the extinction of plankton;
2. Free zooplankton equilibrium point $E_1(c,p,0)$, where $p = \frac{1}{\gamma} \left(\frac{Bc}{c+c_1} - \sigma \right)$, and c is a positive root of the following equation:

$$X_1c^4 + X_2c^3 + X_3c^2 + X_4c + X_5 = 0.$$

Here, $X_1 = -m\gamma$, $X_2 = -\delta(B - \sigma) - m\gamma(c_1 + c_0 + c_2)$, $X_3 = (B - \sigma)(A - \delta)c_0 + \sigma\delta c_1 - m\gamma(c_1c_0 + c_1c_2 + c_0c_2)$, $X_4 = (B - \sigma)Ac_0c_2 - \sigma c_1(A - \delta)c_0 - m\gamma c_0c_1c_2$, and $X_5 = -\sigma Ac_0c_1c_2$.

3. Coexistence equilibrium $E(c,p,z)$, where $p = \frac{\mu + \sqrt{\mu^2 - 4 \left(\frac{\eta c^2 a}{c^2 + c_4^2} - \mu h a \right) (-\mu g)}}{2 \left(\frac{\eta c^2 a}{c^2 + c_4^2} - \mu h a \right)}$, $z = \frac{ahp^2 + p + g}{ap} \left(\frac{Bc}{c+c_1} - \gamma p - \sigma \right)$,

$$\text{and } c = c_4 \sqrt{\frac{\mu(ahp^2 + p + g)}{\eta ap^2 - \mu(ahp^2 + p + g)}}.$$

3.3. Stability Analysis

The stability of the equilibrium points is investigated by evaluating the system's Jacobian matrix at each equilibrium point.

The Jacobian matrix J_0 at $E_0(0,0,0)$ is given by:

$$J_0 = \begin{bmatrix} -m & A & 0 \\ 0 & -\sigma & 0 \\ 0 & 0 & -\mu \end{bmatrix}$$

Here, the eigenvalues are $\lambda_1 = -m$, $\lambda_2 = -\sigma$, and $\lambda_3 = -\mu$. Since all eigenvalues are negative, so E_0 is always locally asymptotically stable.

The Jacobian matrix J_1 at $E_1(c,p,0)$ is given by:

$$J_1 = \begin{bmatrix} -\frac{Ac_0p}{(c+c_0)^2} - \frac{\delta c_2p}{(c+c_2)^2} - m & \frac{mc}{p} & -\frac{vc}{c+c_3} \\ \frac{Bc_1p}{(c+c_1)^2} & -\gamma p & -\frac{ap}{ahp^2 + p + g} \\ 0 & 0 & \left(\frac{\eta c^2}{c^2 + c_4^2} \right) \cdot \frac{ap}{ahp^2 + p + g} - \mu \end{bmatrix}$$

Here, one eigenvalue is $\lambda_1 = \left(\frac{\eta c^2}{c^2 + c_4^2} \right) \cdot \frac{ap}{ahp^2 + p + g} - \mu$, and the other eigenvalue can be obtained by solving the equation:

$$\lambda^2 - Q_1\lambda + Q_2 = 0 \quad (3)$$

where:

$$Q_1 = -\frac{Ac_0p}{(c+c_0)^2} - \frac{\delta c_2p}{(c+c_2)^2} - m - \gamma p < 0 \text{ and}$$

$$Q_2 = \gamma p \left(-\frac{Ac_0p}{(c+c_0)^2} - \frac{\delta c_2p}{(c+c_2)^2} - m \right) - \frac{B_1mc}{(c+c_1)^2} > 0.$$

Therefore, the stability of the equilibrium point $E_3(c,p,0)$ is subsequently determined solely by the sign of the λ_1 .

The Jacobian matrix J at $E(c,p,z)$ is given by:

$$J = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

where:

$$a_{11} = -\frac{Ac_0p}{(c+c_0)^2} - \frac{\delta c_2p}{(c+c_2)^2} - \frac{vc_3z}{(c+c_3)^2} - m < 0,$$

$$a_{12} = -\frac{\delta c}{c+c_2} + \frac{Ac_0}{c+c_0} = \frac{c}{p} \left\{ \frac{vz}{c+c_3} + m \right\} > 0,$$

$$a_{13} = -\frac{vc}{c+c_3} < 0,$$

$$\begin{aligned}
a_{21} &= \frac{Bpc_1}{(c+c_1)^2} > 0, \\
a_{22} &= \frac{Bc}{c+c_1} - 2\gamma p - \sigma - \frac{apz(p+2g)}{(ahp^2+p+g)^2} = -\gamma p - \frac{apz(g-ahp^2)}{(ahp^2+p+g)}, \\
a_{23} &= -\frac{ap^2}{ahp^2+p+g} < 0, \\
a_{31} &= \frac{2\eta c_4^2 c}{(c^2+c_4^2)(ahp^2+p+g)} \cdot \frac{ap^2 z}{(c^2+c_4^2)(ahp^2+p+g)} > 0, \\
a_{32} &= \frac{\eta c_4^2}{c^2+c_4^2} \cdot \frac{apz(p+2g)}{(ahp^2+p+g)^2} > 0, \text{ and} \\
a_{33} &= 0.
\end{aligned}$$

Based on the Routh–Hurwitz criterion, the equilibrium point $E(c, p, z)$ has three eigenvalues with negative real parts if the following conditions are satisfied.

$$a_{22} < 0 \text{ and } a_{11}a_{22} > a_{12}a_{21}$$

3.4. Numerical Simulation

Numerical simulations were conducted to illustrate the dynamic interaction between dissolved oxygen concentration, phytoplankton, and zooplankton based on the mathematical model derived in the previous section. Through this simulation, the behavior of the solutions of the differential equation system representing the interactions among ecosystem components can be observed more clearly over time. To perform the numerical simulations, parameter values representing the biological and ecological characteristics of the modeled system are required. These parameter values were obtained from previous studies published in relevant scientific journals. The parameter values used in this study as follows:

$$\begin{aligned}
c_0 = 1, \quad \delta = 1, \quad c_2 = 1, \quad m = 0.5, \quad B = 1.8, \quad c_1 = 1.0, \quad \gamma = 0.7, \quad \sigma = 0.1, \quad v = 0.01, \\
\mu = 0.01, \quad h = 1.2, \quad a = 3.0, \quad \eta = 0.7, \quad c_4 = 1, \quad g = 0.03, \quad c_3 = 1
\end{aligned}$$

The results of the numerical simulations are then visualized in graphical form to illustrate the dynamic patterns of phytoplankton and zooplankton populations as well as changes in dissolved oxygen concentration within the system. Through the analysis of these simulation results, the stability of the system, the tendency of population growth or decline, and the influence of interactions among variables in the ecological model can be observed.

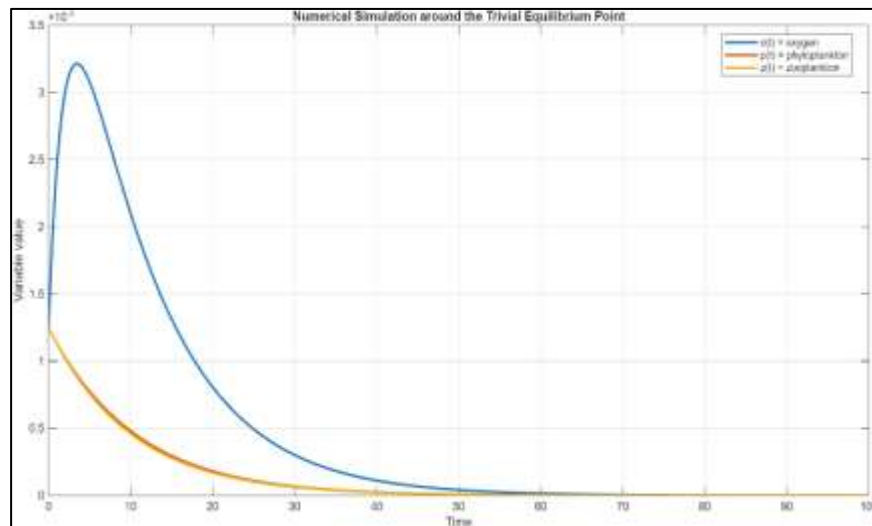


Figure 2. Numerical simulation around the trivial equilibrium point

Figure 2 illustrates the trajectories of the dissolved oxygen concentration $c(t)$, phytoplankton population $p(t)$, and zooplankton population $z(t)$ over time near the trivial equilibrium point. The initial conditions were chosen close to the equilibrium point, namely $c(0) = p(0) = z(0) = 0.00125$. The graph shows that all variables gradually decrease and approach zero as time increases. The oxygen concentration exhibits a slight initial increase before declining exponentially, while the phytoplankton and zooplankton populations decrease monotonically. This behavior indicates that trajectories starting near E_0 converge to the equilibrium point, confirming the local stability of the trivial equilibrium. Ecologically, this state corresponds to an environment in which biological populations cannot be sustained, leading to the collapse of the aquatic ecosystem.

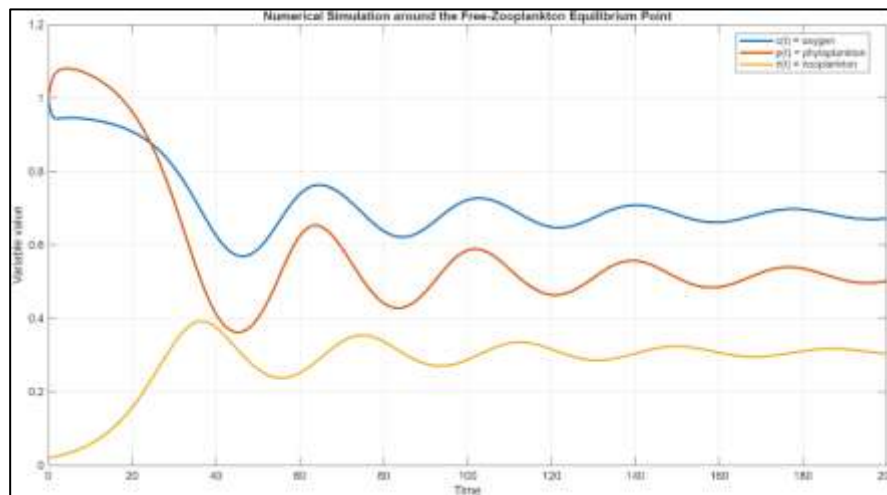


Figure 3. Numerical simulation around the free-zooplankton equilibrium point

Figure 3 shows the dynamic behavior of the dissolved oxygen concentration $c(t)$, phytoplankton population $p(t)$, and zooplankton population $z(t)$ near the zooplankton-free equilibrium point. The initial condition was selected near the equilibrium point to observe the response of the system to small perturbations. The simulation results indicate that the dissolved oxygen concentration and phytoplankton population initially vary dynamically before approaching stable values, whereas the zooplankton population declines toward zero. The oxygen concentration slightly increases at the beginning due to photosynthetic activity of phytoplankton and subsequently decreases as the system approaches equilibrium. These results suggest that the zooplankton-free equilibrium represents a stable ecological condition where phytoplankton can persist without grazing pressure from zooplankton. From an ecological perspective, this condition may occur when environmental factors or disturbances suppress zooplankton survival in the aquatic ecosystem.

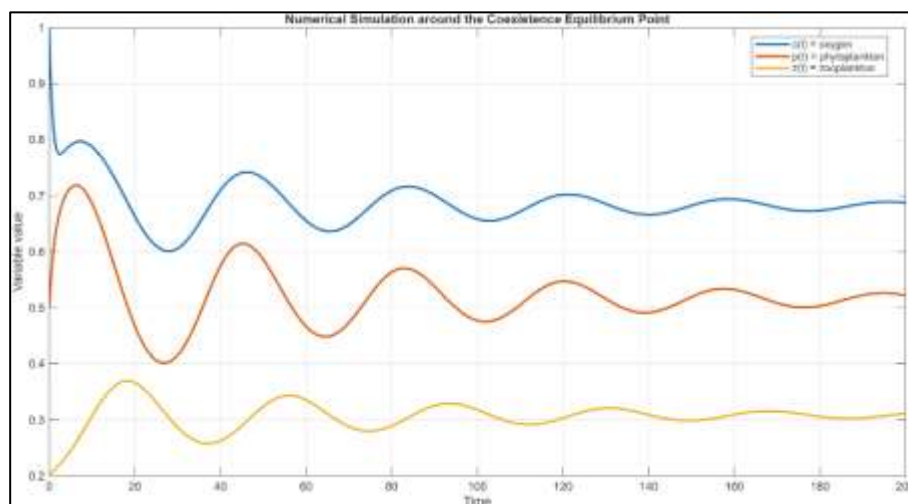


Figure 4. Numerical simulation around the coexistence equilibrium point

Figure 4 illustrates the time evolution of the dissolved oxygen concentration $c(t)$, phytoplankton population $p(t)$, and zooplankton population $z(t)$ near the coexistence equilibrium point. The simulation results show that all variables initially exhibit transient dynamics and subsequently converge to positive steady-state values. The oxygen concentration fluctuates due to the interaction between photosynthetic production and biological consumption, while the phytoplankton and zooplankton populations gradually stabilize over time. This result indicates that the coexistence equilibrium represents a biologically feasible and dynamically stable state in which all ecosystem components can persist. Ecologically, this equilibrium reflects a balanced interaction between oxygen production, phytoplankton growth, and zooplankton predation within the aquatic ecosystem.

4. Conclusions

A mathematical model describing the interaction between dissolved oxygen, phytoplankton, and zooplankton has been developed and analyzed. The system is represented by nonlinear differential equations incorporating ecological processes such as photosynthesis, respiration, and predation. The analysis shows that the system possesses several equilibrium points including trivial and interior equilibria. Stability analysis using the Jacobian matrix and the Routh–

Hurwitz criterion indicates that equilibrium points can be locally asymptotically stable depending on ecological parameters. Numerical simulations support the analytical results and illustrate the dynamic behavior of the ecosystem. The proposed model provides insight into the ecological mechanisms governing plankton–oxygen interactions in aquatic environments.

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Conflict of Interest

The authors declare that he/she has no conflicts of interest to report study.

Declaration of Generative AI and AI-assisted technologies

Generative AI tools were used solely for language refinement (grammar, spelling, and clarity). The scientific content, analysis, interpretation, and conclusions were developed entirely by the authors. The authors reviewed and approved all final text.

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